

New integer programming models for tactical and strategic underground production scheduling

B. King - PhD, Operations Research with Engineering, Colorado School of Mines, Golden, CO, USA

M. Goycoolea – Associate Professor, Business School, Universidad Adolfo Ibáñez, Santiago, Chile

A. Newman – Professor, Mechanical Engineering, Colorado School of Mines, Golden, CO, USA

Abstract:

We consider an underground production scheduling problem which consists of determining the proper time interval(s) in which to complete each mining activity so as to maximize a mine's discounted value, while adhering to precedence, activity durations, and production and processing limits. We present two different integer programming formulations for modeling this optimization problem. Both formulations possess a resource-constrained project scheduling problem structure. The first formulation uses a fine time discretization and is better suited for tactical mine scheduling applications. The second formulation, which uses a coarser time discretization, is better suited for strategic scheduling applications. We illustrate the strengths and weakness of each formulation with examples.

Introduction:

Project scheduling is an important aspect of underground mine planning that consists of determining the start dates for a given set of activities so as to maximize the value of a project, while adhering to operational and resource-availability constraints. Important activities that require scheduling include development, drilling, stoping or other ore-extraction techniques, and backfilling. Precedence relationships impose an order in which activities can be carried out based on their location in the mine. For example, "the activity a associated with development of an area must be completed before the activity a' associated with extraction of that same area can begin." Resources include attributes of the mining operation such as the amount of extraction and mill capacity available per time period, and are determined by capital and equipment availability, among other factors. Correspondingly, for our setting, resource-availability constraints consider the amount of material that can be extracted and sent to the mill (i.e., processed) per time period.

We define the Underground Mine Project Scheduling Problem, or UG-PSP, as that of scheduling a set of mining activities in such a way as to maximize the net present value of the project, while adhering to precedence and resource-availability constraints; in general, optimization models for underground scheduling are more complex than their open pit counterparts (O'Sullivan, Brickey, and Newman, 2015). The UG-PSP is a particular case of the Resource-Constrained Project Scheduling Problem (RCPSP), a class of optimization problems known for their difficulty (Artigues et al., 2008). It should be noted, however, that the UG-PSP may have a multitude of feasible solutions. Many mine planning software packages typically rely on heuristics. In this article, we are concerned with using mixed-integer programming to determine a provably optimal schedule, i.e. the schedule with the highest net present value.

Trout (1995) first proposed a mixed-integer program to solve a 55-stope UG-PSP over a two-year time horizon using multiple time fidelities. The detailed formulation did not gain widespread adoption due to slow solution times. Little et al. (2013) demonstrate the value of implementing scheduling optimization in the mine design process. Others have created case-specific formulations for a variety of underground mines (Carlyle and Eaves, 2001; Nehring et al., 2010; Martinez and Newman, 2011; Epstein et al., 2012). Newman and Kuchta (2007) provide a model for scheduling the Kiruna mine in which activity duration spans multiple time periods; see also Sarin and West-Hansen (2005), O'Sullivan and Newman (2014), and Brickey (2015) for similar models applied to different mines. Little et al. (2011) outline several aggregation techniques to reduce the number of variables a UG-PSP problem contains, while Salama et al. (2015) examine how changing the production rate changes the value of the UG-PSP solution.

UG-PSP Formulations:

We begin by introducing notation for our integer programming (IP) formulations of the UG-PSP, and by noting our assumptions. Our formulations are streamlined, generalized, and highly versatile. That is, they contain precedence and resource constraints, which can be tailored to a specific application, and which are the primary two types of

constraints found in strategic underground models. Specifically, with appropriate rewriting, such a formulation can be applicable to sublevel caving mines (e.g., Kuchta et al., 2004), to block caving mines (e.g., Newman et al., 2013), to sublevel stoping mines (e.g., King et al., 2015), and to cut-and-fill and room-and-pillar mines (O’Sullivan and Newman, 2015). We focus on hard-rock mining methods. Coal extraction would require additional considerations. We next present our two, time-indexed formulations, expressed in the “by” form to improve computational tractability (Lambert et al., 2014).

Sets:

\mathcal{T} : uniform time intervals over which scheduling occurs
 \mathcal{A} : activities available for scheduling
 \mathcal{P}_a : predecessors of activity $a \in \mathcal{A}$
 \mathcal{R} : scarce resources that are consumed

Parameters:

$l_{\bar{a}a}$: number of time intervals that must elapse between the start of activity $a \in \mathcal{A}$ and the start of its predecessor activity $\bar{a} \in \mathcal{P}_a$, referred to as *lag*
 d_a : number of time intervals required to complete activity $a \in \mathcal{A}$ (calculated by rounding up the exact duration to the nearest integer)
 p_{at} : objective function value associated with starting activity $a \in \mathcal{A}$ in time interval $t \in \mathcal{T}$
 q_{ar} : total quantity of resource type $r \in \mathcal{R}$ used to complete activity $a \in \mathcal{A}$
 R_{rt} : total amount of resource type $r \in \mathcal{R}$ available in time interval $t \in \mathcal{T}$

Assumptions:

- A1. In order to begin an activity $a \in \mathcal{A}$, it is necessary to have started *all* activities $\bar{a} \in \mathcal{P}_a$ at least $l_{\bar{a}a}$ time intervals before a . This is not common to all underground production scheduling models. See, for example, O’Sullivan and Newman (2014).
- A2. Once an activity is started, it cannot be interrupted.
- A3. If the duration d_a of an activity $a \in \mathcal{A}$ is greater than one, then the amount of resources consumed per time interval while completing activity a is equal to $\frac{q_{ar}}{d_a}$ for all $r \in \mathcal{R}$.

Note that Assumption A2 can be relaxed for some or all activities in the following way. If preemption is allowed for an activity $a \in \mathcal{A}$ such that $d_a > 1$, then activity a can be replaced by a set of activities $\{a^1, a^2, \dots, a^{d_a}\}$, each with duration one, such that these smaller activities correspond to completing portions of the whole. It is necessary to re-define the precedence relationships and relevant parameters accordingly.

UG-PSP Tactical Formulation:

In the UG-PSP tactical formulation, we construct time intervals that are sufficiently short to capture the detail required to accurately model the duration of underground activities. If a mine schedules activities that require a minimum of one day to complete, a daily fidelity model is appropriate.

Variables:

x_{at} : 1 if activity a is started by time interval t ; 0 otherwise

Objective Function:

$$\text{UG-PSP tactical} \quad \max \sum_{a \in \mathcal{A}} \sum_{t \in \mathcal{T}} p_{at} (x_{at} - x_{a,t-1}) \quad (1)$$

Constraints:

$$x_{a,t-1} \leq x_{at} \quad \forall a \in \mathcal{A}, t \in \mathcal{T} \quad (2)$$

$$x_{at} \leq x_{\bar{a},t-l_{\bar{a}a}} \quad \forall a \in \mathcal{A}, \bar{a} \in \mathcal{P}_a, t \in \mathcal{T} \quad (3)$$

$$\sum_{a \in \mathcal{A}} \frac{q_{ar}}{d_a} (x_{at} - x_{a,t-d_a}) \leq R_{rt} \quad \forall r \in \mathcal{R}, t \in \mathcal{T} \quad (4)$$

$$x_{at} \in \{0,1\} \quad \forall a \in \mathcal{A}, t \in \mathcal{T} \quad (5)$$

The objective function, (1), cumulates the values associated with starting activities in a specified time interval. This may correspond to discounted metal or discounted cash flow. Constraints (2) force a completed activity to remain completed. Constraints (3) enforce precedence relationships. Constraints (4) impose a bound on the resource consumption in each time interval. Constraints (5) restrict all variables to be binary.

The UG-PSP tactical formulation increases in size with the number of time intervals, making instances spanning long horizons using a fine fidelity intractable in practice. In addition, the model assumes detailed knowledge regarding resource and activity attributes. In practice, when planning activities far in the future, these details are difficult to estimate with precision. For these two reasons, the UG-PSP tactical formulation is very well suited for medium-term scheduling, but another model is required for strategic scheduling.

UG-PSP Strategic Formulation:

In the UG-PSP strategic formulation, we approximate the UG-PSP problem by coarsening the time fidelity; this may be advantageous when a finer time fidelity results in a significant number of time periods and a large number of binary variables. Specifically, we create a set of time intervals by aggregating $\Delta \geq 1$ intervals of \mathcal{T} . If an activity $a \in \mathcal{A}$ exists such that $d_a < \Delta$, the resulting model fails to correctly capture the original precedence relationships.

Consider the following example: Suppose that there are eight development activities which occur along the same heading, each corresponding to advancing 5m in a drift (see Figure 1). Each of these activities requires one day to complete and is linked with the appropriate precedence. Aggregation into one-week time intervals, i.e., $\Delta = 7$, results in a lag of zero between consecutive development activities, because the aggregated time intervals are long enough for both the predecessor and successor activity to occur in the same aggregated time interval. If the weekly development capacity is 50m, the aggregated model would allow all eight activities to be completed in a week. This is not possible, because completing all eight activities would require eight days. The aggregated model fails to prevent this infeasibility.

To address this problem, we add precedence relationships: For every pair of activities (a, \hat{a}) that cannot be carried out over the course of Δ consecutive time intervals, we define a precedence relationship such that $\hat{a} \in \hat{\mathcal{P}}_a$ must be completed at least one aggregated time interval in advance of a . We call these *delay precedence relationships*.

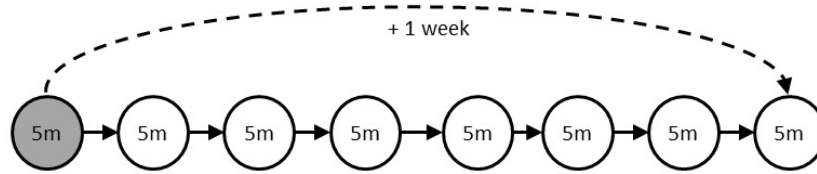


Figure 1: Development activities are represented by nodes, and the precedence relationships are depicted with solid arrows. The selected activity, highlighted in gray, and the successor activity that is 35m away, cannot be completed in the same week, and a delay precedence relationship, shown with dashed arrow, is added.

The resulting model formulation follows.

Sets:

$\overline{\mathcal{T}}$: aggregated time intervals which are Δ time intervals larger than those in \mathcal{T} .

$\hat{\mathcal{P}}_a$: delay predecessors of activity $a \in \mathcal{A}$.

Parameters:

$\bar{l}_{\bar{a}a}$: number of aggregated time intervals that must elapse between the start of activity $a \in \mathcal{A}$ and the start of its predecessor activity $\bar{a} \in \mathcal{P}_a$ (If both a and \bar{a} can be completed in Δ time intervals, let $\bar{l}_{\bar{a}a} = 0$; else, define $\bar{l}_{\bar{a}a} = \lceil \frac{l_{\bar{a}a}}{\Delta} \rceil$.)

\bar{d}_a : number of aggregated time intervals required to complete activity $a \in \mathcal{A}$ (If d_a is not an integer multiple of Δ , the value \bar{d}_a can be obtained by rounding up to the nearest positive integer.)

$\bar{p}_{a\bar{t}}$: objective function value associated with starting activity $a \in \mathcal{A}$ in aggregated time interval $\bar{t} \in \bar{\mathcal{T}}$

$\bar{R}_{r\bar{t}}$: total amount of resource type $r \in \mathcal{R}$ available in aggregated time interval $\bar{t} \in \bar{\mathcal{T}}$

Variables:

$\bar{x}_{a\bar{t}}$: 1 if activity a is started by aggregated time interval \bar{t} ; 0 otherwise

Objective Function:

$$\text{UG-PSP strategic} \quad \max \sum_{a \in \mathcal{A}} \sum_{\bar{t} \in \bar{\mathcal{T}}} \bar{p}_{a\bar{t}} (\bar{x}_{a\bar{t}} - \bar{x}_{a,\bar{t}-1}) \quad (6)$$

Constraints:

$$\bar{x}_{a,\bar{t}-1} \leq \bar{x}_{a\bar{t}} \quad \forall a \in \mathcal{A}, \bar{t} \in \bar{\mathcal{T}} \quad (7)$$

$$\bar{x}_{a\bar{t}} \leq \bar{x}_{\bar{a},\bar{t}-\bar{t}_{\bar{a}a}} \quad \forall a \in \mathcal{A}, \bar{a} \in \mathcal{P}_a, \bar{t} \in \bar{\mathcal{T}} \quad (8)$$

$$\bar{x}_{a\bar{t}} \leq \bar{x}_{\hat{a},\bar{t}-1} \quad \forall a \in \mathcal{A}, \hat{a} \in \hat{\mathcal{P}}_a, \bar{t} \in \bar{\mathcal{T}} \quad (9)$$

$$\sum_{a \in \mathcal{A}} \frac{q_{ar}}{\bar{d}_a} (\bar{x}_{a\bar{t}} - \bar{x}_{a,\bar{t}-1}) \leq \bar{R}_{r\bar{t}} \quad \forall r \in \mathcal{R}, \bar{t} \in \bar{\mathcal{T}} \quad (10)$$

$$\bar{x}_{a\bar{t}} \in \{0,1\} \quad \forall a \in \mathcal{A}, \bar{t} \in \bar{\mathcal{T}} \quad (11)$$

The objective function, (6), cumulates the value associated with starting an activity in a specified time interval. Constraints (7) force a completed activity to remain completed. Constraints (8) enforce precedence and constraints (9) force the pair of activities contained in the delay precedence relationship to occur in different time intervals. Constraints (10) bound resource consumption in each time interval. Constraints (11) restrict all variables to be binary.

Two important complications arise when using the UG-PSP strategic formulation. The first is that the number of delay precedence relationships grows rapidly as Δ increases. The second is that a feasible solution in this formulation might not necessarily correspond to a solution that is feasible in the UG-PSP tactical formulation. This is the same limitation suffered by integer programming formulations typically used in open pit production scheduling (Johnson, 1968). As such, this formulation is well suited to scheduling large time horizons and making strategic decisions.

Computational Examples:

We compare the performance of the two formulations using data from two open stoping mines: (i) a small, artificial one and (ii) a real-world mine. While both of these examples use open stoping as the mining method, as we note earlier, the formulations are sufficiently general to incorporate other mining methods as well.

Data:

The single-segment data set is a small section of an underground open stoping mine containing four stopes. In order to extract a stope, the necessary development, stope drilling, and backfilling must have already been completed. Table 1 provides an outline of the activities and their attributes that are used, and Figure 2 outlines the precedence structure and activities in the single-segment data set.

Table 1: Summary of activity characteristics in the underground mine data set. Resource attributes are given as the total resource consumed. The delay column represents the number of days that must pass after the activity is completed before its successor activity can begin.

Activity Type	Quantity	Cost/Profit	Activity Rate	Duration (days)	Delay (days)	Total Resource Consumed by Activity Type		
						Mine Tonnes	Development Meters	Drill Meters

Horizontal Drift	2	-\$10,000/m	5 m/day	10	0	2500	50	0
Cross-Cut	8	\$100/t	5 m/day	4	0	1000	20	0
Stope Drilling	4	-\$100/m	120 m/day	6	0	0	0	720
Stope Mucking	4	\$250/t	500 t/day	10	1	5000	0	0
Backfilling	4	-\$5/t	1000 t/day	4	7	4000	0	0

Note: The lag parameter is the sum of the predecessor's duration and the delay between the predecessor and successor.

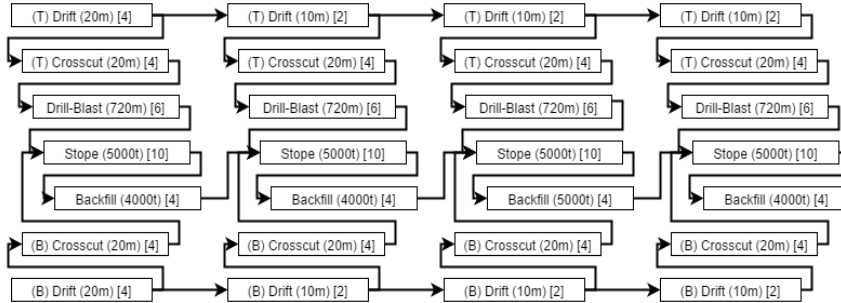


Figure 2: Precedence structure for the underground mine (solid arrows). Naming convention: (T-B) indicates whether the activity is located on the top or bottom level of the stope, followed by the activity type, primary resource consumed ‘()’, and duration of the activity ‘[]’.

We also create two larger data sets by copying the single-segment data set. The “triple” and “penta” data sets consist of three and five copies of the single-segment data set, respectively. The triple and penta stopes are differentiated by their stope values: 100%, 80%, and 60%, and 100%, 90%, 80%, 70%, and 60% of the original values, respectively.

The UG-PSP tactical formulation is modeled at daily time fidelity, and the UG-PSP strategic formulation uses an aggregated 14-day time fidelity. The UG-PSP strategic formulation activities are disaggregated, for example, into 10 stoping activities that each require one day, contain 500 ore tons, and are appropriately linked with precedence; delay constraints are constructed using the disaggregated activities. Daily production limits are 15 meters of development, 1000 tonnes of total extraction, 1000 tonnes of backfilling, and 240 meters of drilling. The discount rate for all models and data sets is 0.10% for every 14 days, or a daily discount rate of 0.0683%, and the objective is to maximize NPV. Note that although, in practice, different discount rates might apply to models of varying time fidelity, in this case, we normalize the discount rates to allow for a comparison between the objective function values and solutions of the two models.

Numerical Results:

The UG-PSP tactical and UG-PSP strategic formulations for the single-segment, triple, and penta data sets are coded in the algebraic modeling language AMPL (AMPL 20140908, 2015) and solved to the default optimality tolerance using CPLEX 12.6.0.1 (IBM CPLEX optimizer, 2015) on a Dell PowerEdge R410 machine with 16 processors (2.72 GHz each) and 28 GB of RAM. Table 2 provides a summary of the different models and the solution times for each formulation and data set.

Table 2: Summary of problem size, objective function value, and solution times for the various model formulations and problem instances.

Formulation	Number of Variables	Number of Resource Constraints	Number of Precedence and Delay Constraints	Objective Function Value (NPV)	Solution Time (sec)	Time Horizon (days)
UG-PSP tactical (Single)	2,660	380	3,135	2,901,017	0.08	95
UG-PSP strategic (Single)	1,099	28	2,359	2,918,144	0.29	98
UG-PSP tactical (Triple)	10,166	468	11,415	6,196,407	86.8	115
UG-PSP strategic (Triple)	4,308	32	9,099	6,308,911	7.06	126
UG-PSP tactical (Penta)	26,740	764	31,515	8,891,858	13,817.74	191
UG-PSP strategic (Penta)	10,990	56	23,590	9,062,961	99.57	196

The key differences between the UG-PSP tactical and the UG-PSP strategic formulations are the objective function value, solution time, and solution, i.e., schedule. We observe that the NPV is slightly different for a given data set between the two formulations. The UG-PSP strategic always has a higher optimal NPV than that of the UG-PSP tactical, because the UG-PSP strategic formulation is an approximation of UG-PSP. (The discount rate calculations are tied to the time fidelity of the model.) Nonetheless, the NPV difference between the two formulations for the same data set is never greater than 2%, and although the two formulations produce slightly different NPVs, the overall extraction quantities resulting from the solution of both UG-PSP models are very similar. Figure 3 shows the production tonnes associated with the solution for the triple data set using both formulations, demonstrating the similarities in output.

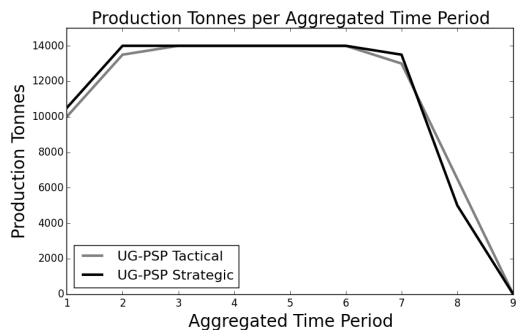


Figure 3: Tonnes extracted in every aggregated time period, i.e., 14 days. The production tonnage for the UG-PSP tactical formulation is calculated by summing the production tonnage in each 14-day interval.

The solution time for each formulation also varies drastically; for example, the penta data set using the UG-PSP tactical formulation requires 13,817 sec., which is much longer than the solution time for the UG-PSP strategic formulation, 99.57 sec. The UG-PSP tactical formulation does not scale well; solution times for single, triple, and penta data sets are 0.08, 86.8, and 13,817.74 seconds, respectively. Figure 4 demonstrates the change in the number of variables and solution time as a function of time horizon length.

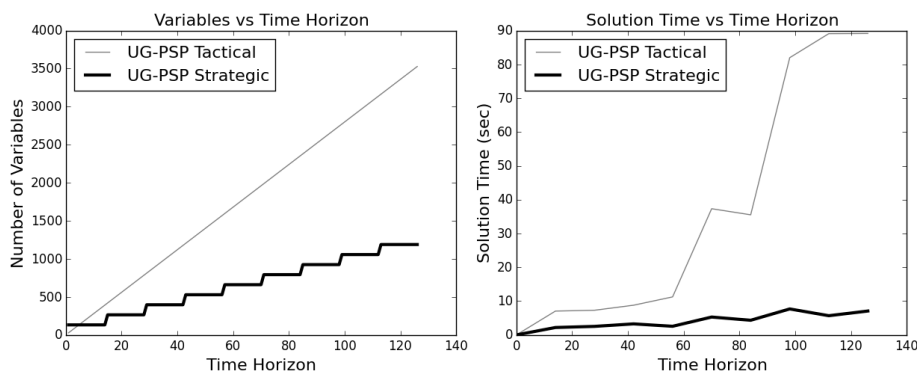


Figure 4: The number of variables, and solution times for the UG-PSP tactical and UG-PSP

Although the UG-PSP strategic formulation results in much smaller instances than the equivalent tactical ones, the strategic formulation produces solutions with respect to aggregated time periods, rather than with the fidelity of the more detailed and directly implementable schedules from the UG-PSP tactical formulation. The activities to be completed in one aggregated time period in the UG-PSP strategic model may not be feasibly completed with respect to the UG-PSP tactical formulation over the same time period; for example, production may be significantly skewed towards the beginning or the end of an aggregated time period.

To illustrate how the formulations perform in practice, we apply both the UG-PSP tactical (using daily fidelity) and strategic (using 60-day fidelity) formulations to a dataset from a large underground stoping mine that contains over 25,000 activities and resource constraints that limit development, stope extraction, backfilling, and pollutant

emissions. Setting the optimality gap at 5% (to compensate for the large problem instances), we obtain relative solution times that qualitatively closely match those of the synthetic dataset (see Table 3).

Table 3: Solutions times for a real-world underground mining data set with thousands of activities. Note: *indicates that CPLEX is unable to find a solution within 5% of optimality in 72 hours.

Time Horizon (days)	UG-PSP Tactical Solution Time	UG-PSP Strategic Solution Time
60	00h 00m 59s	00h 00m 01s
120	68h 05m 07s	00h 00m 01s
180	*	00h 00m 06s
360	*	00h 05m 13s
720	*	43h 32m 40s

Conclusion:

The contribution of this paper lies in our distinction between two formulations for the UG-PSP; while the tactical formulation contains more detail, it doesn't scale well with horizon length. Therefore, for longer time horizons, we recommend our strategic formulation. We use commercially available software and both small and larger instances to demonstrate. Additional real-world examples appear in Brickey (2015) and King et al. (2015); however, these authors use a specialized algorithm (Bienstock and Zuckerberg, 2010) with enhancements (Chicoisne et al., 2012; Muñoz et al., 2015) to solve their problem instances. This highlights the fact that the formulations discussed in this paper can be exploited so as to solve instances large enough to be of interest and importance to the mining industry.

Acknowledgements:

Alexandra Newman and Barry King received funding from the Center for Innovation in Earth Resource Science and Engineering at the Colorado School of Mines, and from Alford Mining Systems. Marcos Goycoolea received funding from FONDECYT grant #1151098 and CONICYT PIA Anillo grant 1407.

References

- AMPL, 2015, Version 20140908, www.ampl.com.
- Artigues, C., Demasse, S., and Neron, E. 2008, *Resource-Constrained Project Scheduling: Models, Algorithms, Extensions and Applications*. Wiley-ISTE, pp 288.
- Bienstock, D., and Zuckerberg, M., 2010, "Solving LP relaxations of large-scale precedence constrained problems," *Integer Programming and Combinatorial Optimization*, Vol. 6080, No. 4, pp. 1-14.
- Brickey, A., 2015, "Underground Production Scheduling Optimization with Ventilation," *Ph.D Thesis*, Colorado School of Mines, Golden, Colorado, pp. 80.
- Carlyle, M., and Eaves, C., 2001, "Underground planning at Stillwater Mining Company," *Interfaces*, Vol. 31, No. 4, pp. 50-60.
- Chicoisne, R., Espinoza, D., Goycoolea, M., Moreno, E., and Rubio, E., 2012, "A new algorithm for the open-pit mine production scheduling problem," *Operations Research*, Vol. 60, No. 3, pp. 517-528.
- Epstein, R., Goic, M., Weintraub, A., Catalàn, J., Santibàñez, P., Urrutia, R., Cancino, R., Gaete, S., Aquayo, A., Felipe C., 2012, "Optimizing long-term production plans in underground and open-pit copper mines," *Operations Research*, Vol. 60, No. 1, pp. 4-17.

IBM CPLEX optimizer, 2015, *Version 12.6.0.1*, <http://www-01.ibm.com/software/commerce/optimization/cplex-optimizer/>.

Johnson, T., 1968, "Optimum Open Pit Mine Production Scheduling," *Ph.D. Thesis*, University of California, Berkeley, California, pp. 248.

King, B., Goycoolea, M., and Newman, A., 2015, "Optimizing the open pit-to-underground mining transition," *Under revision with European Journal of Operational Research*.

Kuchta, M., A. Newman, and E. Topal, 2004. "Implementing a Production Schedule at LKAB's Kiruna Mine," *Interfaces*, Vol. 34, No. 2, pp. 124-134.

Lambert, B., Brickey, A., Newman, A., and Eurek, K., 2014, "Open pit block sequencing formulations: A tutorial," *Interfaces*, Vol. 44, No. 2, pp. 127-142.

Little, J., Knights P., and Topal E., 2013, "Integrated optimization of underground mine design and scheduling," *Journal of the Southern African Institute of Mining and Metallurgy*, Vol 113, No. 10, pp. 775-785.

Little, J., and Topal, E., 2011, "Strategies to assist in obtaining an optimal solution for an underground mine planning problem using Mixed Integer Programming," *International Journal of Mining and Mineral Engineering*, Vol 3, No. 2, pp. 152-172.

Martinez, M., and Newman, A., 2011, "A solution approach for optimizing long- and short-term production scheduling at LKAB's Kiruna Mine," *European Journal of Operational Research*, Vol. 211, No. 1, pp. 184-197.

Muñoz, G., Espinoza, D., Goycoolea, M., Moreno, E., Queyranne, M., and Rivera, O., 2015, "Production scheduling for strategic open pit mine planning, part I: A mixed integer programming approach," *Working Paper*.

Nehring, M., Topal, E., and Little, J., 2010, "A new mathematical programming model for production schedule optimization in underground mining operations," *Journal of the South African Institute of Mining & Metallurgy*, Vol 110, No. 8, pp. 437-446.

Newman, A., and Kuchta, M., 2007, "Using aggregation to optimize long-term production planning at an underground mine," *European Journal of Operational Research*, Vol. 176, No. 2, pp. 184-197.

Newman, A., C. Yano, and E. Rubio, 2013. "Mining Above and Below Ground: Timing the Transition," *IIE Transactions*, Vol. 45, No. 8, pp. 865-882.

O'Sullivan, D., and Newman, A., 2014, "Extraction and backfill scheduling in a complex underground mine," *Interfaces*, Vol. 44, No. 2, pp. 204-221.

O'Sullivan, D., Brickey, A., and Newman, A., 2015, "Is openpit production scheduling "easier" than its underground counterpart?," *Mining Engineering*, Vol. 67, No. 4, pp. 68-73.

Salama, A., Nehring, M., and Jenny, G., 2016, "Analysis of the impact of increasing mining rate in underground mining using simulation and mixed integer programming," *International Journal of Mining Science and Technology*.

Sarin, S. C., and West-Hansen, J., 2005, "The long-term mine production scheduling problem," *IIE Transactions*, Vol 37, No. 2, pp. 109-121.

Trout, L. P., 1995, "Underground mine production scheduling using mixed integer programming," *25th International APCOM Symposium Proceedings*, July 9-14, Brisbane, Australia, pp. 395-400.