Linear Programming Approximations for Modeling Instant-Mixing Stockpiles

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ABSTRACT: Stockpiles are a crucial part of mine planning. However, they are often ignored in long-term planning due to the difficulty of correctly evaluating their impact in mine scheduling. This difficulty arises mainly because materials of different grades are mixed in a stockpile, and the final grade of the material leaving the stockpile is a complex non-linear function of the material inside the stockpile. In practice, computational software uses different (usually linear) approximations for estimating this grade, but it is not clear how good these approximations are.

In this paper, we discuss different optimization models to approximate the real impact of a stockpile on long-term mine planning. We discuss the properties of these models and compare the quality of the approximations computationally. We show that it is possible to obtain good upper and lower bounds on the resulting grade of the stockpile, and realistic and accurate estimations of the behavior of the stockpile. We also discuss how to extend these models to address different minerals and their corresponding grades.

INTRODUCTION

In recent years, several authors have proposed new methodologies to solve integer programming models for the open pit production-scheduling problem (see Newman et al., 2010). These types of models have been revamped due to the possibility of solving them efficiently, even for large-scale real instances (Goycoolea et al. 2013; Muñoz et al., 2014). Nevertheless, the inclusion of stockpiles in these models has been avoided, due to the difficulty of correctly modeling the mixing behavior of the material inside a stockpile. Because materials of different grades are mixed in a stockpile, the final grade of the material leaving the stockpile becomes a complex non-linear function of the material inside the stockpile. Bley et al. (2009) have proposed a quadratic model for production scheduling that assumes "instant-mixing" inside the stockpile, that is, that all grades of material inside the stockpile are averaged. However, this type of model is computationally very difficult to solve, limiting its use in real instances. Instead, commercial software relies on basic approximations of this non-linear behavior. In this paper, we discuss these models, and we propose a new model to approximate the non-linear behavior of a single stockpile. To simplify this study, we assume that the extraction schedule has already been decided, and we are able to choose only the destination (processing, stockpiling or dumping) of each block.

NON-LINEAR MATHEMATICAL MODELING INSTANT-MIXING STOCKPILES

In this section, we present the adapted models proposed by Bley et al. (2009) for the production-scheduling problem with stockpiles. Let B be a set of blocks, each with a total ore tonnage w_b and a total metal tonnage m_b, for each b in B. Hence, the *grade* of block b is given by m_b/w_b. We are interested in determining the destination of these blocks over a set of T periods, each period with a processing capacity of C_t. To simplify the problem, we assume that each block b has a prescheduled extraction time t_b, and we denote the set of blocks extracted at time t by B_t. For the objective function, we compute the net present value (NPV) at a discount rate of δ, considering incomes and costs. The incomes are equal to v for each ton of metal processed, and we expend a processing cost of c^p for each ton processed and a handling cost c^s for each ton moved from the stockpile to the processing plant.

A first model proposed by Bley et al. (2009) is presented in Figure 1, referred to as the *warehouse* model. In this model, the variables $x^p{}_b$ and $x^s{}_b$ represent the fraction of block b sent directly to the processing plant and to the stockpile, respectively. The variables $z^s{}_{b,t}$ represent the fraction of block b in the stockpile at the end of period t, and variables $z^p{}_{b,t}$ represent the fraction of block b sent to the processing plant from the stockpile during period t. Finally, the variables f_t represent the fraction of each sent from the stockpile to the processing plant.

The objective function (1.1) maximizes the NPV, which measures revenues minus costs. Eq. (1.2) indicates that the fractions of a block sent to processing and the stockpile sum to less than or equal to 1. Eq. (1.3) computes the fraction of block b in the stockpile at the end of period t, which is either the fraction of the block sent to the stockpile at the time of extraction ($x^s{}_b$) or the fraction from previous period minus the fraction sent to the processing plant at time t. The capacity constraint of the processing plant at each period is modeled by Eq. (1.4). Finally, Eq. (1.5) requires that the fraction of a block that is currently at the stockpile sent to processing is the same for all

$$\max \sum_{t\in\mathcal{T}} \left(\frac{1}{1+\delta}\right)^t \left(v \cdot \left(\sum_{b\in\mathcal{B}_t} m_b x_b^p + \sum_{b\in\mathcal{B}} m_b z_{b,t}^p\right) \right. \tag{1.1}$$

$$\left. - c^p \cdot \sum_{b\in\mathcal{B}_t} w_b x_b^p - (c^p + c^s) \cdot \sum_{b\in\mathcal{B}} w_b z_{b,t}^p \right)$$

$$x_b^p + x_b^s \leq 1 \qquad \forall b \in \mathcal{B} \tag{1.2}$$

$$z_{b,t}^s = \begin{cases} z_{b,t-1}^s - z_{b,t}^p & \text{if } t > t_b \\ x_b^s & \text{if } t = t_b \end{cases} \qquad \forall b \in \mathcal{B}, t \in \mathcal{T} \tag{1.3}$$

$$\sum_{b\in\mathcal{B}_t} w_b x_b^p + \sum_{b\in\mathcal{B}} w_b z_{b,t}^p \leq C^t \qquad \forall t \in \mathcal{T} \tag{1.4}$$

$$\frac{z_{b,t}^p}{z_{b,t}^s + z_{b,t}^p} = f_t \qquad \forall b \in \mathcal{B}, t \in \mathcal{T} \tag{1.5}$$

$$x_b^p, x_b^s, z_{b,t}^p, z_{b,t}^s, f_t \geq 0 \qquad \forall b \in \mathcal{B}, t \in \mathcal{T} \tag{1.6}$$

Figure 1. Warehouse model for an instant-mixing stockpile (Bley et al., 2009)

$$\max \sum_{t\in\mathcal{T}} \left(\frac{1}{1+\delta}\right)^t \left(v\cdot\left(\sum_{b\in\mathcal{B}_t} m_b x_b^p + a_t^p\right)\right. \tag{2.1}$$

$$\left. -c^p \cdot \sum_{b\in\mathcal{B}_t} w_t x_b^p - (c^p + c^s)\cdot o_t^p\right)$$

$$x_b^p + x_b^s \leq 1 \qquad \forall b \in \mathcal{B} \tag{2.2}$$

$$o_t^s = \begin{cases} \sum\limits_{b\in B_t} w_b x_b^s & \text{if } t = 1 \\ o_{t-1}^s - o_t^p + \sum\limits_{b\in B_t} w_b x_b^s & \text{if } t > 1 \end{cases} \qquad \forall t \in \mathcal{T} \tag{2.3}$$

$$a_t^s = \begin{cases} \sum\limits_{b\in B_t} m_b x_b^s & \text{if } t = 1 \\ a_{t-1}^s - a_t^p + \sum\limits_{b\in B_t} m_b x_b^s & \text{if } t > 1 \end{cases} \qquad \forall t \in \mathcal{T} \tag{2.4}$$

$$\sum_{b\in\mathcal{B}_t} w_b x_b^p + o_t^p \leq C^t \qquad \forall t \in \mathcal{T} \tag{2.5}$$

$$\frac{a_t^p}{o_t^p} = \frac{a_{t-1}^s}{o_{t-1}^s} \qquad \forall t \in \mathcal{T} \tag{2.6}$$

$$x_b^p, x_b^s, a_t^p, a_t^s, o_t^p, o_t^s \geq 0 \qquad \forall b \in \mathcal{B}, t \in \mathcal{T} \tag{2.7}$$

Figure 2. Basic model for an instant-mixing stockpile (Bley et al., 2009)

blocks that are currently in the stockpile. This last constraint ensures that the grade of the material sent to the stockpile is equal to the average grade of all blocks in the stockpile.

As explained in Bley et al. (2009), an equivalent model can be obtained by aggregating variables $z^p_{b,t}$ and $z^s_{b,t}$ into the total metal and ore tonnage in the stockpile. For this purpose, we replace these variables with new variables o^p_t and a^p_t, representing the total tonnage of ore and metal sent from the stockpile to the processing plant at time t, respectively. Equivalently, the new variables o^s_t and a^s_t represent the total tonnage of ore and metal remaining in the stockpile at the end of period t. The resulting model is presented in Figure 2, and they refer to this as the *basic* model.

Eqs. (2.1), (2.2) and (2.5) correspond to the former Eqs. (1.1), (1.2) and (1.4). Eqs. (2.2) and (2.3) represent the total ore and metal available in the stockpile at the end of time t, respectively. Finally, Eq. (2.6) requires the grade of the material sent to the processing plant from the stockpile at time t to be equal to the grade of the material remaining in the stockpile at the end of time t-1.

Note that both constraints (1.5) and (2.6) are non-linear, non-convex constraints, making these problems very difficult to solve, even for small instances. For more details on computational approaches for solving these models, see Bley et al. (2012).

LINEAR MODELS FOR OBTAINING UPPER AND LOWER BOUNDS FOR THE PROBLEM

Because these non-linear models are, from a computational point of view, unviable to solve for realistically sized instances, linear models are required to approximate this non-linear behavior and provide bounds on the corresponding objective function value.

An upper bound can be obtained from the warehouse formulation in Figure 1 by removing the non-linear constraint (1.5). That is, we assume that a block can be sent to the stockpile and be processed in a future period, independently of other blocks (and their grades) sent to or from the stockpile. Note that this solution is not feasible for the original problem, but it provides an upper bound on the objective of the problem.

As a second approach to obtain a feasible lower bound, refer to the basic formulation in Figure 2. Let L be a threshold for the grade of the stockpile and assume that the material going to the stockpile has a grade of at least L, and that the material leaving the stockpile for processing has a grade equal to L. This behavior can be obtained from the basic model by removing the variables a^s_t, flow constraints (2.4), and the non-linear constraint (2.6) and adding the following constraints:

$$a^p_t = L \cdot o^p_t \qquad \forall t \in \mathcal{T} \tag{3.1}$$

$$x^s_b = 0 \qquad \forall b \in \mathcal{B} : \frac{m_b}{w_b} < L \tag{3.2}$$

The first constraint requires that the metal sent to processing have grade L, and the second constraint forbids blocks with grade less than L to be sent to the stockpile. Note that for a given solution of this model, it is possible to construct a solution for the basic formulation, with the same value for variables x^p_b, x^s_b, o^p_t and o^s_t and a higher value for a^p_t in each period, hence obtaining a solution for the basic formulation with a higher objective value. Thus, the proposed model effectively produces a lower bound on the objective value of the non-linear model for any value of L. We call this model the *L-bound* model.

A third approach to obtaining a better lower bound for the problem is derived from the previous linear model: instead of forbidding sending blocks with grade less than L to the stockpile, we force that the cumulative average grade of the material sent to the stockpile to be at least L. This can be achieved as in the previous formulation, replacing constraint (3.2) by:

$$\sum_{s \le t} \sum_{b \in B_s} m_b x^s_b \ge L \cdot \sum_{s \le t} \sum_{b \in B_s} w_b x^s_b \quad \forall t \in \mathcal{T} \tag{4}$$

We call this model the *L-average* model. Note that any solution of the *L-bound* model is also feasible for the *L-average* model, so the latter provides a better bound of the non-linear problem.

COMPUTATIONAL RESULTS

Our first experiment compares the quality of the bounds obtained by the different linear models with the non-linear model. We used the data from the Marvin instance (53,271 blocks) available at the Minelib website, using the extraction schedule from the best-known feasible solution for this instance. The economic parameters for this instance appear in the Minelib website (http://mansci.uai.cl/minelib), and we added a re-handling cost c^s of 0.3 \$/ton. Additionally, we solved the instance for reduced processing capacities (namely, 60%, 70%, 80% and 90% of the original capacity) to make the use of stockpiles more profitable. For the instant-mixing behavior, we used the warehouse formulation and solved it using SCIP 3.1.0 (Achterberg 2009). The solutions presented correspond to the best solution after 60 hours of computation, which are not optimal but have an optimality gap less than 2%. Some of these instances required up to 200 GB of memory to be solved. By contrast, the linear models can be solved in a few seconds on a laptop computer. For the L-bound and L-average models, we present the results for the best value of L found.

The objective value of the non-linear model and the difference between objectives with the other models are presented in Table 1. A first result is that, for this instance, the upper bound is very close to the objective of the instant-mixing model, but it does not provide a feasible solution for the problem. To obtain a feasible solution, the L-average model obtains an objective value considerably closer to the real value than the L-bound model. In fact, for three of the five cases, the L-average model obtains the optimal value of the instant-mixing model. Note that for this problem, the

Table 1. Comparison of the different models for Marvin instance

Capacity	UB	Instant-mixing	L-Average	L-Bound	No Stock	L-Average (value of L)	L-Bound (value of L)
60%	+2.1%	742,292 M$	–0.3%	–4.9%	–11.8%	0.77	0.73
70%	+1.3%	820,693 M$	–0.1%	–3.8%	–8.1%	0.66	0.65
80%	+0.6%	882,863 M$	–0.0%	–2.5%	–5.1%	0.60	0.60
90%	+0.3%	928,833 M$	–0.0%	–1.4%	–2.9%	0.55	0.55
100%	+0.1%	961.253 M$	–0.0%	–0.7%	–1.3%	0.50	0.50

Table 2. Results for an instance with blending constraints

Upper bound	4,848,040 M$
L-Average model	4,669,790 M$
L-Bound model	4,451,700 M$
No Stockpile	4,296,550 M$

stockpile is used to store profitable material that cannot be processed until a period after its extraction when enough capacity becomes available.

Our second numerical experiment corresponds to a mine with blending constraints. The instance is a copper-gold mine with a high content of arsenic. In addition to processing capacity, there is a blending constraint on the average grade of arsenic of the material sent to processing. Hence, the stockpile is mostly used for storing and blending material from different periods to satisfy this environmental constraint. The model is larger than our previous instance (~200,000 blocks scheduled over 50 years), so it was not possible to solve the non-linear model for this problem. To obtain a feasible solution, we set an upper bound on the grade of arsenic for the stockpile as well as a lower bound on the grade of the metal.

The objective values obtained by each model are presented in Table 2. Note that the potential impact of using a stockpile in this case is higher than in the previous numerical experiment. As before, the *L*-average model improves the objective value, obtaining a better feasible solution than the *L*-bound model.

In Figure 3, we present the metal grade and the arsenic grade of the material going from the mine and the stockpile to the processing plant, for the *L*-average model (up) and the Upper Bound (down) model. It can be seen that effectively the upper bound doesn't provide a feasible solution. In fact, the grades of the material from the stockpiles differ considerably from one period to the next, showing that there is no instant-mixing property in the solution. On the other hand, the *L*-average solution obtains a more realistic result, showing that it is closer to the instant-mixing property of the non-linear model.

CONCLUSIONS

We present three linear models used for upper and lower approximations of the non-linear formulation of the production-scheduling problem for an instant-mixing stockpile. Computational studies show that the L-average model obtains a better approximation of the non-linear behavior of a stockpile that is close to optimal. Hence, it is appropriate for using on more complex production-scheduling problems where the extraction time is decided considering the existence of a stockpile.

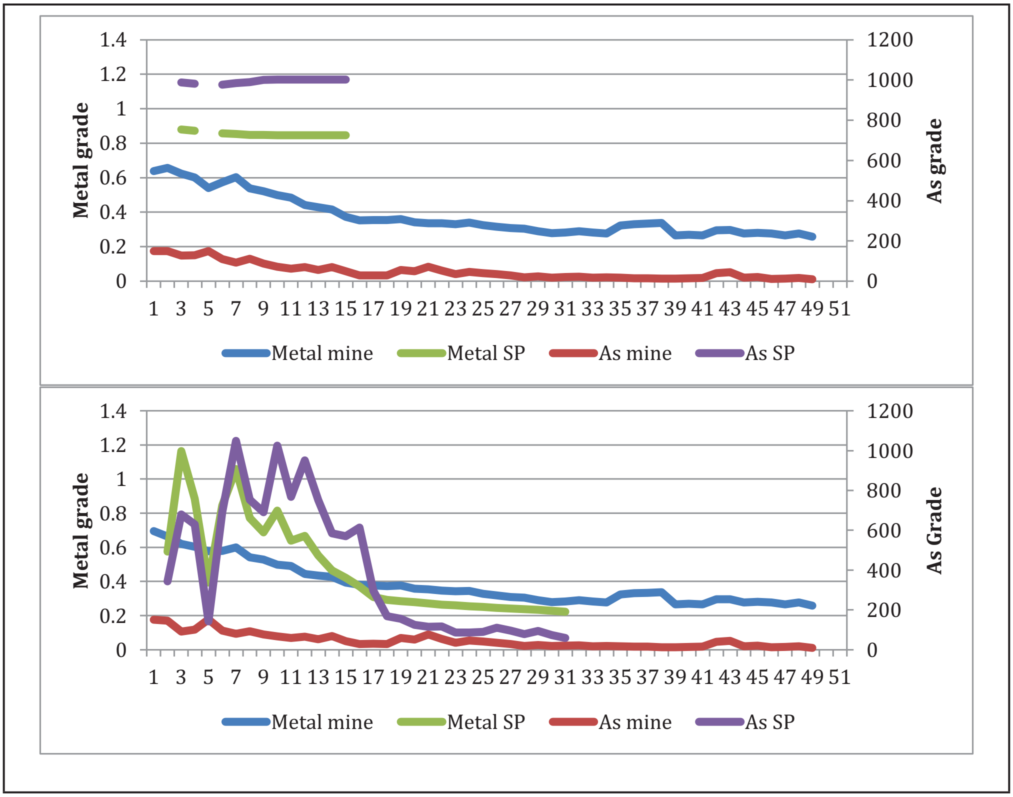

Figure 3. Ore grade for the *L*-average (up) and Upper bound (down) solution in each period

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