OPEN PIT MINE SCHEDULING UNDER UNCERTAINTY: A ROBUST APPROACH

Daniel Espinoza, Assistant Professor, Department of Industrial Engineering, Universidad de Chile, Santiago, Chile, 8370439, daespino@dii.uchile.cl.

Marcos Goycoolea, Associate Professor, School of Business, Universidad Adolfo Ibañez, Santiago, Chile, 7941169, marcos.goycoolea@uai.cl.

Eduardo Moreno*, Associate Professor, Faculty of Engineering and Sciences, Universidad Adolfo Ibañez, Santiago, Chile, 7941169, eduardo.moreno@uai.cl (corresponding author).

Gonzalo Muñoz, PhD Student, Department of Industrial Engineering and Operations Research, Columbia University, New York, New York, USA, 10027, gm2543@columbia.edu.

Maurice Queyranne, Professor, Sauder School of Business, University of British Columbia,Vancouver, Canada, V6R1T3, maurice.queyranne@sauder.ubc.ca.

ABSTRACT

In order to carry out an open pit mining operation, planners must periodically prepare what is known as strategic mine plan. This is a tentative production schedule for the remaining life of the mine that defines which area of a mining reserve will be extracted, in what years this extraction will take place, which resources will be used for the extraction, and how the extracted material will be treated or processed. Given a discretized representation of the mining reserve, the problem of actually computing such a production schedule is known as the Open Pit Mine Production Scheduling Problem (OPM-PSP).

OPM-PSP is widely acknowledged to be one of the most critical parts of a mine planning effort: Not only is it instrumental for investors looking to understand the expected cash flows of a project, but also, strategic decisions resulting from solving OPM-PSP can have binding consequences in the life of a mining project.

An important limitation of traditional methodologies for solving OPM-PSP is that they fail to explicitly address the volatility of metal prices. In fact, these approaches typically assume a long-term fixed price for each metal, when in truth future prices are unknown.

Though it is known that mine planning solutions are very sensitive to price volatility, we are not aware of any attempt to quantify this sensitivity, nor that try to deal with it.

In this study, a mean-reverting stochastic process for modeling ore-price uncertainty is proposed. This model allows to analyze the sensitivity of mine planning solutions obtained by traditional mine planning optimization methods. Computational results confirm that solutions are extremely sensitive, and quantify the extent to which small price perturbations can result in tremendous losses of profits.

Secondly, this paper pursues to determine if mine planning optimization methods can be modified (by explicitly taking into account price volatility) in order to produce solutions that

are less sensitive. To this end, a robust optimization method is implemented and the solutions obtained are compared to those obtained by the traditional methods.

Computational results suggest that there do not exist robust solutions that afford much protection against price volatility, thus raising the question: How should price volatility be dealt with in practice?

INTRODUCTION

Open pit mines are typically represented by a three-dimensional discretization of the ore-body made up of equal sized units known as blocks. In such a block model, geological attributes such as tonnage, grade, rock hardness, location etc. are assigned to individual blocks (Hustrulid and Kuchta, 2006).

Given a set of pre-determined resources available for mining it is possible to define for each block one or several destinations or treatment options, including sending the block to a waste-dump, mill, leech-pad, stockpile or other destination. For each block it is possible to estimate the cost of sending it to any given destination, as well as the corresponding amount of recovered metals.

Given a block model, the Open Pit Mine Production Scheduling Problem (OPM-PSP) formally consists in deciding which blocks should be extracted, when they should be extracted and what should be done with them (e.g., send to waste dump, processing plant, stock piles etc.) in such a way as to maximize profits with the available resources, and comply with operational constraints.

The first known formal model for describing this problem dates back to Johnson (1968), who proposes an integer-programming formulation and describes a decomposition method for solving it. Since then a number of methodologies have been proposed for solving Johnson's model. Among others, these include the works of Dagdelen and Johnson (1986), Boland et al (2009), Bienstock and Zuckerberg (2010) and Chicoisne et al (2012).

An important limitation of traditional methodologies for solving OPM-PSP is that they fail to explicitly address the volatility of metal prices. In fact, these approaches typically assume a long-term fixed price for each metal, when in truth future prices are unknown.

There is very little work on addressing price uncertainty in open-pit mine production scheduling problems. Recent work includes that of Abdel and Dimitrakopolous (2011). A closely related problem that has been more commonly addressed is that of geologic uncertainty. For a background on this topic see Dimitrakopolous (2011) and Vielma et al (2009). One desired framework to deal with uncertainty in the ore prices is robust optimization. Kumral (2010) presents a robust optimization model to deal with uncertainty in prices. However, this approach is limited by the size of the block model, and can only deal with a small number of prices' scenarios.

This paper introduces a robust framework for open-pit production scheduling. Using this framework two publicly available mine planning problems are analyzed. This analysis shows that the proposed approach can be used to quantitatively measure the sensitivity of solutions to price volatility. In fact, the analysis shows that for both problem instances, the optimal value is extremely sensitive to small price perturbations. What is more alarming is that the analysis shows that there do not exist solutions that are significantly more robust. That is, it is not possible to modify the solution so that small price fluctuations have such an adverse effect on objective function value.

METHODOLOGY

A deterministic model for the open pit mine production-scheduling problem

This section describes an integer programming formulation of the open pit mine production-scheduling problem. It starts by describing the data and parameters, and follow-up with the actual formulation.

Sets:

B the set of all blocks.

A the set of all precedence relationships: (a,b) is in A if block a must be extracted no later than block b.

D the set of destinations to which a block could be sent (dump, mill, leech plant etc.).

M the set of minerals in the ore-body that could be processed.

$T = \{1,...,t_{max}\}$ the set of time periods under consideration.

Constants:

$c_{b,d,t}$ the discounted cost of sending block b to destination d in time t.

$q_{b,d,m,t}$ the amount of mineral m recovered by sending block b to destination d in time t.

$p_{m,t}$ the discounted price of mineral m in time t.

U , u a matrix and a vector with the same number of rows. This matrix includes additional constraints on the processed blocks.

Variables:

$x_{b,t}$ binary variable indicating whether block b should be extracted in time t.

$y_{b,d,t}$ binary variable indicating whether block b should be sent to destination d in time t.

$z_{m,t}$ continuous variable indicating the amount of mineral m recovered in time t.

Definition. *Given the sets, constants and variables described above, the Open Pit Mine Production Scheduling Problem (OPM-PSP) is the following linear program:*

$$\text{(OPM-PSP):} \quad \begin{array}{l} \min c\,y - p\,z \\ (x, y, z) \in F \end{array}$$

where,

$$F = \begin{cases} (x,y,z): & \begin{array}{ll} x_{b,t} = \sum\limits_{d \in D} y_{b,d,t} & \text{for all } b \in B, \ t \in T \quad (1) \\ z_{m,t} = \sum\limits_{b \in B, d \in D} q_{b,d,m,t} y_{b,d,t} & \text{for all } m \in M, \ t \in T \quad (2) \\ \sum\limits_{t=1}^{\tau} x_{b,t} \le \sum\limits_{t=1}^{\tau} x_{a,t} & \text{for all } (a,b) \in A, \ \tau \in T \quad (3) \\ Uy \le u & \quad (4) \\ x_{b,t}, y_{b,d,t} \in \{0,1\} & \text{for all } b \in B, d \in D, t \in T \quad (5) \end{array} \end{cases}$$

Note that in this formulation, (1) imposes that each block should be sent to exactly one destination, (2) defines the z variables, (3) imposes the precedence relationship between blocks, (4) represents all other operational constraints that can be modeled using the y variables (typically mineral-processing constraints, mine-extraction constraints, blending constraints etc.), and (5) imposes integrality of the x,y variables.

OPM-PSP is equivalent to the PCPSP formulation of Bienstock and Zuckerberg (2010), except for the fact additional variables $z_{m,t}$ are introduced, which model the amount of mineral m recovered in time period t. As described later, this difference is important in modeling mineral price uncertainty.

A probabilistic model of price uncertainty

In OPM-PSP it is assumed that the price of a mineral commodity m in time period t is known and has value p_t^m. That is, for each mineral commodity m it is assumed to know a discrete-time deterministic time series

$$p^m = (p_1^m, p_2^m, \dots, p_{t_{max}}^m),$$

describing the evolution of the price of mineral m over time. A much better model would assume p^m to be random. In order to revise model OPM-PSP and explicitly incorporate price uncertainty it is first necessary to adopt an adequate probabilistic model for the description of p^m. Such a model should be sufficiently complex to reasonably capture the stochastic nature of commodity prices, and yet be sufficiently simple to be tractable for an optimization model. Such a model is described in this section. Since the same model is applied to all minerals, the index m is dropped in the rest of this section.

First consider a continuous stochastic variable $p(t)$ representing the price of a mineral in time t. A very common assumption used when modeling stochastic processes of the price of commodities such as copper, gold and other natural resources, is that $p(t)$ follows an Arithmetic Ornstein-Uhlenbeck process (Dixit and Pindyck, 1994). This process is described by the stochastic differential equation (6) (Oksendal, 2010),

$$dp = \eta(\bar{p} - p)dt + \sigma_{OU}dz \qquad (6)$$

where η and $\bar{p}$ are non-negative scalars corresponding to the speed of reversion and the mean of the process, respectively, and where σ_{OU} corresponds to the standard-deviation of the white-noise. What this equation essentially describes is a stochastic process $p(t)$ that can fluctuate in the short term, but that in the long term will be drawn back towards the mean projected value $\bar{p}$. A discrete-time version of process $p(t)$ is an AR(1) process described by the following first-order auto-regressive process (Dixit and Pindyck, 1994), and it's given in equation (7):

$$p_t = \bar{p}(1 - e^{-\eta}) + e^{-\eta}p_{t-1} + \epsilon_t, \qquad (7)$$

where the terms ϵ_t correspond to independently and normally distributed random variables with mean zero and standard deviation

$$\sigma = \sqrt{\frac{\sigma_{OU}{}^2}{2\eta}(1 - e^{-2\eta})}.$$

The vector

$$p = (p_1, p_2, \dots, p_{t_{max}}),$$

corresponding to the solution of recursion (7), is a natural discrete-time stochastic time series model of the price of minerals. In fact, it can be seen from (7) that in each time period t, the value p_t is obtained from p_{t-1} by shifting it towards $\bar{p}$, and introducing some random "white noise" (represented by ϵ_t). Not only is (7) the natural discretization of the continuous Ornstein-Uhlenbeck process, but also it is quite tractable. By substituting out the value of p_{t-1} in recursion (7) it is not difficult to show that

$$p_t = \bar{p}\,(1 - e^{-\eta})(\textstyle\sum_{i=0}^{t-1} e^{-i\eta}) + e^{-t\eta}p_0 + \sum_{i=0}^{t-1} e^{-i\eta}\epsilon_{t-i}.$$

Note that mineral prices p_t are constructed as a linear function of normally distributed random variables ϵ_i. Hence, each random variable p_t follows a normal distribution with mean

$$\mathbb{E}(p_t) = \bar{p} + e^{-t\eta}(p_0 - \bar{p}),$$

and a covariance between prices in two periods s and t given by

$$Cov(p_s, p_t) = \sigma^2 e^{-|s-t|\eta}\frac{1-e^{-2t\eta}}{1-e^{-2\eta}}.$$

It follows that it is reasonable to model vector p^m, for each mineral m, as a multivariate normal distribution. Moreover, if parameters $\eta^m, \bar{p}^m$, and p_0^m associated to the underlying continuous Ornstein-Uhlenbeck process can be estimated, then it is possible to explicitly compute the expected value of p^m, henceforth $\mu^m = \mathbb{E}(p^m)$, and the covariance matrix, henceforth C^m. Note that in this analysis the discount factor has been omitted. If this discount is applied, then $\mu_t^m = \frac{1}{(1+\tau)^t}\mathbb{E}(p_t^m)$ and $C_{st}^m = \frac{1}{(1+\tau)^{s+t}}Cov(p_s, p_t)$, where τ is the discount factor.

For a more thorough background on modeling commodity ore prices with an arithmetic Ornstein-Uhlenbeck process, see Dixit and Pindyck (1994).

For each mineral m consider a stochastic discrete-time series vector p^m representing the price of m over time. For each value $\alpha > 0$, consider an ellipsoid P_m^α, centered in μ^m, defined as follows:

$$P_m^\alpha = \{p : p = \mu^m + S_m w, \ \|w\| \le \alpha\}.$$

where S_m is the square-root of matrix C^m. Observe that these ellipsoids are nested in the sense that if $\alpha_1 < \alpha_2$, then $P_m^{\alpha_1} \subseteq P_m^{\alpha_2}$. Thus, for any given $\varepsilon > 0$ it is possible to determine α such that $\mathrm{Prob}(p^m \in P_m^\alpha) = \varepsilon$. In fact, this can be achieved by defining $\alpha = \sqrt{F_{t_{max}}^{-1}(\varepsilon)}$, where $F_n^{-1}(x)$ represents the inverse cumulative distribution function of the Chi-squared distribution with n degrees of freedom.
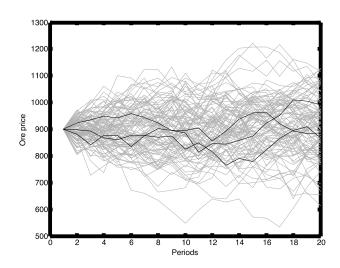
Case study: Analyzing the impact of price uncertainty on two problems.

In order to evaluate the impact of price uncertainty on a mine planning solution, two publicly available instances (Espinoza et al, 2012) of OMP-PSP (Marvin and McLaughlin_limit) are studied. Basic information describing each of these instances is presented in Table 1.

Table 1 - Mine instances. Further details in Minelib website (http://mansci.uai.cl/minelib).

Mine Name	Mineral	# Blocks	Max # Periods	Metal prices
Marvin	Au, Cu	53,271	20 years	12 \$/gr, 2000 \$/ton
McLaughlin_limit	Au	112,687	30 years	900 \$/oz

Each of these two instances has a deterministic price time series $p^m = p_0^m$ associated to each corresponding metal. In order to conduct this study it is assumed that these prices are, instead, stochastic variables p^m, following a multivariate normal distribution with mean $\mathbb{E}(p^m) = \mu^m = p_0^m$. It is assumed that this distribution corresponds to an Ornstein-Uhlenbeck process with reversion speed $\eta = 0.1$, and with standard deviation of the white noise equal to $\sigma = \mu^m/50$. These parameters produce a standard deviation of ore price after 20 years of approximately a 5% of the expected ore price. Figure 1 illustrates 100 samples of price time series using the multivariate normal distribution resulting from the described parameters for the McLaughlin_limit instance.

Figure 1 - 100 price time-series sampled for the McLaughlin_limit metal data sets. The time-series in dark-black are in the ε =1% ellipsoid.

To evaluate the impact of uncertainty, note that given a fixed set of vectors z_m, for all $\alpha > 0$, it can be computed the following conservative measure (see Ben-Tal and Nemirovski, 2001):

$$\min_{p^m \in P_m^\alpha} \sum_m p^m z_m = \sum_{m \in M} \mu^m z_m - \alpha \sum_{m \in M} \|S_m z_m\|_2$$

That is, given a solution (x,y,z) of OPM-PSP, and given an ellipsoid P_m^α, this problem compute the worst-possible price $p^m \in P_m^\alpha$, in the sense that p^m achieves the worst-possible objective function value of all prices in P_m^α for (x,y,z).

Next, define $\alpha_i = \sqrt{F_{t_{max}}^{-1}(\varepsilon_i)}$, for i = 1,2,3, where $\varepsilon_1 = 0.01, \varepsilon_2 = 0.5$ and $\varepsilon_3 = 0.9$.

Table 2 compare the objective function value of the optimal solution considering that the price of each metal p^m is deterministic and equal to p_0^m, to the objective function obtained assuming that p^m is the worst-possible price in $P_m^{\alpha_i}$ for i = 1,2,3. This can be interpreted as follows: What is the worst that can happen if the price p^m is different than p_0^m by a little-bit (low level of uncertainty; $\varepsilon = 0.01$), by a medium amount (medium level of uncertainty; $\varepsilon = 0.5$) and a high amount (high level of uncertainty; $\varepsilon = 0.9$)? Results are presented relative to the average case. That is, relative to the solution obtained by solving the deterministic problem, and evaluating the objective function value in the mean price.

Table 2 – NPV value of the optimal mine plan, relative to the value of the deterministic case, in different worst-case scenarios.

Instances	Average case	Uncertainty		
		Low	**Medium**	**High**

		$(\varepsilon = 0.01)$	$(\varepsilon = 0.5)$	$(\varepsilon = 0.9)$
Marvin	1.000	0.823	0.730	0.673
McLaughlin_limit	1.000	0.913	0.878	0.857

What Table 2 shows is that solutions are very sensitive to metal price fluctuations, and that the worst that can happen is pretty bad. In fact, it can be seen that even under low uncertainty, the NPV drops by 17.7% and 8.7% for Marvin and McLaughlin_limit, respectively. Under high uncertainty these values can drop even more, by 32.7% and 14.3% respectively.

The high sensitivity is, in great part, due to the fact that margins are very tight. That is, the objective function is of the form pz-cy (income minus costs), and the values of cy and pz are not so different from each other.

A robust model for the open pit mine production-scheduling problem

In order to analyze how much room there is to deal with the difficulties presented in the previous section, the robust counterpart of the problems in question can be solved. That is, instead of maximizing the profits obtained by using an expected price, it is maximized the worst-possible profits that could be obtained if all prices in an uncertainty set are considered. This can be defined formally as follows.

Definition. *Given an instance of OPM-PSP, and sets P_m^α as defined above, the Robust Open Pit Mine Production Scheduling Problem (R-OPM-PSP) consists in solving:*

$$\text{(R-OPM-PSP):} \quad \begin{array}{c} \min f_\alpha(x,y,z) \\ (x,y,z) \in F \end{array}$$

where

$$f_\alpha(x,y,z) = \begin{array}{c} \min c\,y - \sum_m p^m z_m \\ p^m \in P_m^\alpha \quad \forall m. \end{array}$$

In other words, given a schedule (x,y,z), the problem look for the worst price scenario inside the ellipsoid P_m^α for that schedule. It is easy to see that, if $\alpha = 0$, then R-OPM-PSP = OPM-PSP. On the other-hand, for $\alpha > 0$, the optimal value $\bar{v}$ and the optimal solution $(\bar{x}, \bar{y}, \bar{z})$ of R-OPM-PSP will be robust in the sense of Ben-Tal and Nemirovski (2001). Intuitively, this means that the objective function value of R-OPM-PSP is protected against perturbations of the mineral prices p^m provided that these prices remain within its ellipsoid P_m^α.

Lemma 2. *Problem R-OPM-PSP is equivalent to the following Second Order Cone program:*

$$\min c\, y - \sum_{m \in M} w_m$$
$$w_m \leq \mu^m z_m - \alpha \|S_m z_m\| \qquad \text{for all } m \text{ in } M \qquad (8)$$
$$(x, y, z) \in F$$

Proof. See Ben-Tal and Nemirovski (2001).

The Robust Bienstock-Zuckerberg algorithm

Solving R-OPM-PSP may become difficult in practice. In fact, even solving OPM-PSP can be difficult, since practical instances of OPM-PSP tend to be very large in terms of the number of variables and constraints. A typical instance can easily be defined from millions of blocks, up to fifty time periods, multiple possible destinations for each block, and two or three side-constraints per time period (Bienstock and Zuckerberg 2010; Chicoisne et al, 2012). A number of decomposition algorithms have been proposed over the years for solving such instances of OPM-PSP. These range from Dantzig-Wolf decomposition methods (Johnson, 1968), Lagrangian Relaxation methods (Dagdelen and Johnson, 1986), specialized methods for a fixed number of destinations or side-constraints (Boland et al 2009, Chicoisne et al, 2012), and general-purpose methods such as the Bienstock-Zuckerberg decomposition (Bienstock and Zuckerberg, 2010). All of these methods have been shown to work reasonably well on real instances of OPM-PSP. In the authors' experience, the best performing methodology to date is that of Bienstock and Zuckerberg (2010).

A recent and interesting development is that of Muñoz (2012), who shows that it is possible to directly generalize the decomposition method of Bienstock and Zuckerberg (2010) so that it can solve instances of R-OPM-PSP. For this, Muñoz considers the following definition, and proves the following two lemmas.

Definition. *Given a directed graph $G = (N,A)$ with n vertices, and a second-order conic system of inequalities $Du \preccurlyeq d$, with m rows on n variables, the General Second Order Conic Precedence Constrained Problem (GCPCP) is the following second order conic programming program:*

$$\begin{aligned} \min c\, u \\ Du \preccurlyeq d \\ \text{(GCPCP)} \qquad u_i - u_j \leq 0 \quad \forall (i,j) \in A \\ 0 \leq u_i \leq 1 \quad \forall i \in N \end{aligned}$$

Lemma 3. *The continuous relaxation of any instance of ROPM-PSP can be reduced to an equivalent instance of GCPCP with the same number of variables and constraints.*

Lemma 4. *Problem GCPCP can be solved with the Lagrangian algorithm of Bienstock and Zuckerberg (2010) in a finite number of iterations.*

Proof (Lemmas 2 and 3). See Muñoz (2012).

Thus, the Lagrangian algorithm of Bienstock and Zuckerberg may be used to solve the continuous relaxation of R-OPM-PSP. To obtain a feasible integer solution from a fractional solution of the continuous relaxation, Muñoz (2012) shows in a number of instances that the TopoSort and local-improvement heuristics described in Chicoisne et al (2012) work very well.

RESULTS & DISCUSSION

The implementation was developed in the C programming language using CPLEX v12.4. All runs were done in a cluster made up of 10 servers, running in CentOS Linux v5.5, each with 16 GB of RAM. Each run took around 15 minutes.

Each instance is solved three times, using R-OPM-PSP formulation with the low, medium and high uncertainty sets. Table 3 presents two objective function values for each of the solutions obtained. The "Average" objective function value is computed using the mean prices (i.e. evaluated with the fixed metal price of Table 1), and the "Worst" objective function value is the actual objective function of the R-OPM-PSP instance. All results are presented relative to the optimal solutions presented in Table 1, which are referred as the nominal solutions.

Table 3 – Normalized NPV of the robust solution in the average case and in the worst case under different levels of uncertainty.

Instances	Uncertainty					
	Low		Medium		High	
	Average	Worst	Average	Worst	Average	Worst
Marvin	0.996	0.826	0.989	0.737	0.979	0.684
McLaughlin_limit	0.998	0.915	0.997	0.881	0.996	0.861

As should be expected, all of the "Average" solution objective function values are strictly less than those of the nominal solutions (they are less than one). However, it can be seen that the "price of robustness" (or the amount by which the objective function value decreases from one) is low. For both Marvin and McLaughlin, the price of robustness is under a percentage point for the low uncertainty set (0.4 and 0.2 percent, respectively). In the case of the high uncertainty set, the price of robustness is 2,1 and 0.4 percent, respectively, which is still low.

How much is gained from using robust solutions as opposed to the nominal solutions? To quantify this it is necessary to compare values between Table 1 and Table 2. Consider, for instance, the Marvin data set on the Low uncertainty set. Table 1 shows an objective function value of 0.823, whereas in Table 2, an objective function value of 0.826 is observed. That is, the solution value obtained in the worst possible case in the low uncertainty set increased by only

0.3% when the robust solution is used. In order to obtain that improvement it is necessary to sacrifice 0.4% in the mean case. In general, the gains afforded by R-OPM-PSP are low in all of the instances. The best improvement is obtained by the Marvin instance in the high uncertainty set, with an increase in objective function value from 0.673 to 0.684 (a 1.1% increase) using the robust solution, at a cost of 2.1% in the expected objective function value.

Overall, the results are not very encouraging. It is observed that even small price perturbations (within the low uncertainty set) can result in very large objective function value drops (17.7% and 8.7% for Marvin and McLaughling, respectively), and using robust solutions does relatively little to curve these drops (17.4% and 8.5% for Marvin and McLaughlin, respectively). That is to say, solutions are extremely sensitive to prices, and moreover, there do not seem to exist robust solutions that afford a substantial level of protection.
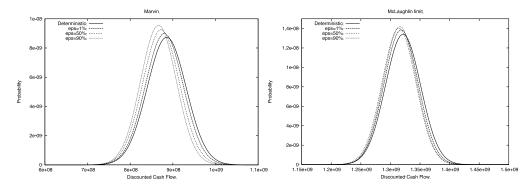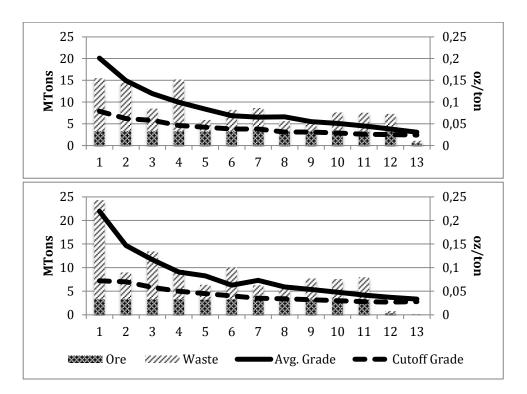

Figure 2 – Probability distribution of the NPV for each solution.

In order to understand how the optimizer is actually changing the schedule in order to make solutions more robust, consider Table 4 presenting the tonnages extracted, cut-off grades and life of mine for each optimal solution. It can be seen that under higher uncertainty, the robust solution increases the average cut-off grade of the ore, thus resulting in a shorter life of mine. Figure 3 compares the yearly tonnage and cut-off grades for McLaughlin_limit of the nominal solution and the robust solution under high uncertainty. It can be seen that the change in the average cutoff grades are more important in the first years, where the economical impact is higher on the NPV.

Table 4 – Tonnages and average grades of the deterministic and robust solutions.

Marvin	Total tonnage	Total ore	Avg Au grad	Avg Cu grade
Deterministic	527.1 Mton	265.1 Mton	0.570 ppm	0.610 %
Robust small unc.	519.7 Mton	258.2 Mton	0.577 ppm	0.617 %
Robust medium unc.	510.7 Mton	251.5 Mton	0.584 ppm	0.623 %
Robust high unc.	498.9 Mton	245.9 Mton	0.588 ppm	0.628 %

McLaughlin_limit	Total tonnage	Total ore	Avg Au grade
Deterministic	111.1 Mton	40.0 Mton	0.0862 oz/ton
Robust small unc.	111.1 Mton	38.4 Mton	0.0877 oz/ton
Robust medium unc.	111.1 Mton	37.8 Mton	0.0882 oz/ton
Robust high unc.	111.1 Mton	36.6 Mton	0.0898 oz/ton

Figure 3 – McLaughlin results for the deterministic case (up) and the robust case under high uncertainty (down).

CONCLUSIONS

- Mine planning solutions are very sensitive to price volatility. This paper provides a mathematical framework for quantitatively measuring this sensitivity and analysing its impact on real mine planning problems. Computational results suggests that even small changes in prices can result in tremendous differences in profits. It seems clear that price volatility is an important issue that should be further studied and better incorporated in mine planning methodologies.

- Robust optimization provides a tool to deal with metal price volatility. Robust optimization works by maximizing the value of the worst possible solution in an uncertainty set. It is known that Robust counterparts of linear programming models can be solved using Second Order Conic programming. In the specific case of R-OPM-PSP, it is possible to adapt the algorithm of Bienstock and Zuckerberg, originally designed for OPM-PSP, to solve R-OPM-PSP. This paper shows that despite the fact the resulting R-OPM-PSP instances can be extremely large, the Bienstock-Zuckerberg algorithm performs very well solving them.

- It is empirically observed that the solutions of R-OPM-PSP tend to have a shorter life of mine than the solutions of OPM-PSP due to the use of higher cutoff grades (particularly on the first periods), and tend to result in smaller final pits. Increasing the level of uncertainty exacerbates these effects.

- Computational results show that robust solutions obtained by solving R-OPM-PSP do not afford much protection in worst-case scenarios, even under small price perturbations. This suggests that in the context of open pit mine planning there is no such thing as a very robust solution.

- Other stochastic optimization models should be studied to deal with metal price uncertainty. A multi-stage stochastic programming approach that considers recourse is much more likely to produce reasonable solutions. Though these solutions will suffer from the same inherent problems observed in this framework (very high losses due to small price perturbations), these solutions will allow planners to obtain a more realistic assessment of what a strategic plan is worth, in terms of NPV. What these results essentially show is that the high level of volatility make the mean price computed for a solution rather meaningless.

ACKNOWLEDGEMENTS

REFERENCES

Abdel, S.A. and Dimitrakopoulos, R. (2011). Incorporating geological and market uncertainties and operational flexibility into open pit mine design. Journal of Mining Science. v. 47. n. 2. pp 191-201.

Ben-Tal, A. and Nemirovski, A. (2001). Lecture notes on modern convex optimization: analysis, algorithms, and engineering applications. Philadelphia, PA. Society for Industrial and Applied Mathematics.

Bienstock, D., and Zuckerberg, M. (2010). Solving LP relaxations of large-scale precedence constrained problems. In Integer Programming and Combinatorial Optimization. Heidelberg, Berlin. Springer. Edited by Friedrich Eisenbrand and Bruce Shepherd. pp. 1-14.

Boland, N., Dumitrescu, I., and Froyland G. (2008). A multistage stochastic programming approach to open pit mine production scheduling under uncertain geology. Optimization Online. Available on http://www.optimization-online.org/DB_HTML/2008/10/2123.html.

Boland, N., Dumitrescu ,I., and Froyland, G. (2009). LP-based disaggregation approaches to solving the open pit mining production scheduling problem with block processing selectivity. Computers and Operations Research. v. 36. n. 4. pp. 1064-1089.

Chicoisne, R., Espinoza, D., Goycoolea, M., Moreno, E., and Rubio, E. (2012). A new algorithm for the open-pit mine production scheduling problem. Operations Research. v. 60. Issue 3. pp. 517-528.

Dagdelen, K., and Johnson, T. (1986). Optimum open pit mine production scheduling by lagrangian parameterization. Proceedings of the 19th APCOM Symposium of the Society of Mining Engineers. pp 127-142.

Dimitrakopolous, R. (2011). Strategic Mine Planning Under Uncertainty. Journal of Mining Science. v. 47. n. 2. pp 138-150.

Dixit, A., and Pindyck, M. (1994). Investment under uncertainty. Princeton. Princeton University Press.

Espinoza, D., Goycoolea, M., Moreno, E., and Newman, A. (2012). MineLib: a library of open pit mining problems. Annals of Operations Research. v. 206. n. 1. pp. 93-114.

Hustrulid, W. A., and Kuchta, M. (2006). Open pit mine planning and design (Vol. 1). London, UK. Taylor & Francis.

Johnson, T. B. (1968). Optimum open pit mine production scheduling (No. ORC-68-11). Doctoral Thesis, Berkeley. USA Operations Research Center.

Kumral, M. (2010). Robust stochastic mine production scheduling, Engineering Optimization. v. 42. n. 6. pp. 567-579.

Muñoz, G. (2012). Modelos de optimizacion lineal entera y aplicaciones a la minería. Mathematical Engineering Thesis. Santiago.Dpt. Math. Engineering, Universidad de Chile.

Oksendal, B. (2010). Stochastic differential equations. Heidelberg , Berlin. Springer.

Vielma, J.P., Espinoza, D., and Moreno. (2009). E. Risk control in ultimate pits using conditional simulations. Proceedings of the 34th International Symposium on Applications of Computers and perations Research in the Mineral Industry. APCOM 2009. pp 107-114.